CALCULUS

Second Edition

BRIGGS
COCHRAN
GILLETT

ALGEBRA

Exponents and Radicals

$x^a x^b = x^{a+b}$ $\qquad \dfrac{x^a}{x^b} = x^{a-b}$ $\qquad x^{-a} = \dfrac{1}{x^a}$ $\qquad (x^a)^b = x^{ab}$ $\qquad \left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$

$x^{1/n} = \sqrt[n]{x}$ $\qquad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$ $\qquad \sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$ $\qquad \sqrt[n]{x/y} = \sqrt[n]{x}/\sqrt[n]{y}$

Factoring Formulas

$a^2 - b^2 = (a - b)(a + b)$ $\qquad a^2 + b^2$ does not factor over real numbers.

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ $\qquad a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$

Binomials

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + b^n,$$

where $\dbinom{n}{k} = \dfrac{n(n-1)(n-2)\cdots(n-k+1)}{k(k-1)(k-2)\cdots 3\cdot 2\cdot 1} = \dfrac{n!}{k!(n-k)!}$

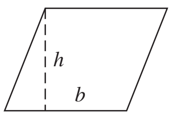
Quadratic Formula

The solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

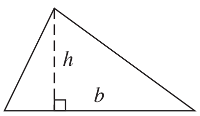
GEOMETRY

Parallelogram



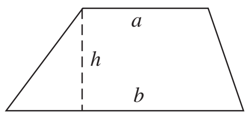
$A = bh$

Triangle



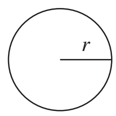
$A = \frac{1}{2}bh$

Trapezoid



$A = \frac{1}{2}(a + b)h$

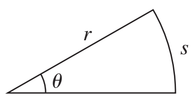
Circle



$A = \pi r^2$

$C = 2\pi r$

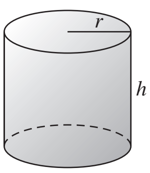
Sector



$A = \frac{1}{2}r^2\theta$

$s = r\theta$ (θ in radians)

Cylinder

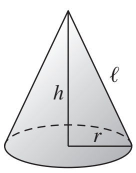


$V = \pi r^2 h$

$S = 2\pi rh$

(lateral surface area)

Cone

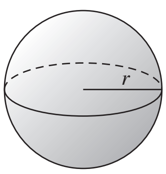


$V = \frac{1}{3}\pi r^2 h$

$S = \pi r\ell$

(lateral surface area)

Sphere

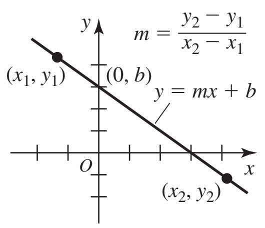


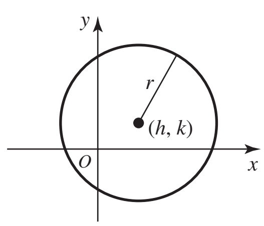
$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Equations of Lines and Circles

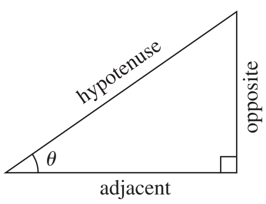
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	slope of line through (x_1, y_1) and (x_2, y_2)
$y - y_1 = m(x - x_1)$	point–slope form of line through (x_1, y_1) with slope m
$y = mx + b$	slope–intercept form of line with slope m and y-intercept $(0, b)$
$(x - h)^2 + (y - k)^2 = r^2$	circle of radius r with center (h, k)





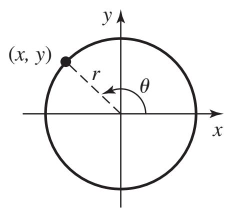
$(x - h)^2 + (y - k)^2 = r^2$

TRIGONOMETRY



$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\qquad \sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\qquad \tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ $\qquad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ $\qquad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

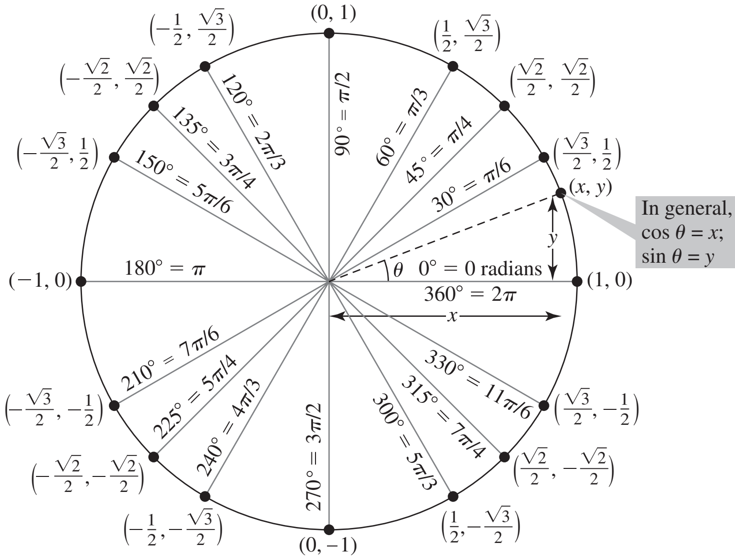


$\cos\theta = \dfrac{x}{r}$ $\qquad \sec\theta = \dfrac{r}{x}$

$\sin\theta = \dfrac{y}{r}$ $\qquad \csc\theta = \dfrac{r}{y}$

$\tan\theta = \dfrac{y}{x}$ $\qquad \cot\theta = \dfrac{x}{y}$

(*Continued*)



Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \quad \tan^2\theta + 1 = \sec^2\theta \quad 1 + \cot^2\theta = \csc^2\theta$$

Sign Identities

$$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta \quad \sec(-\theta) = \sec\theta \quad \cot(-\theta) = -\cot\theta$$

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas

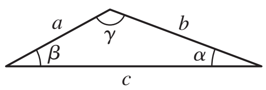
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

Addition Formulas

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Law of Sines

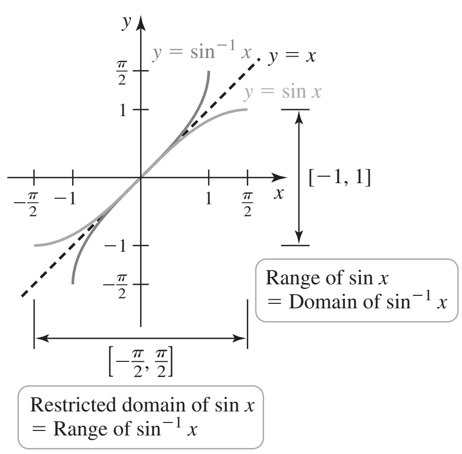
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

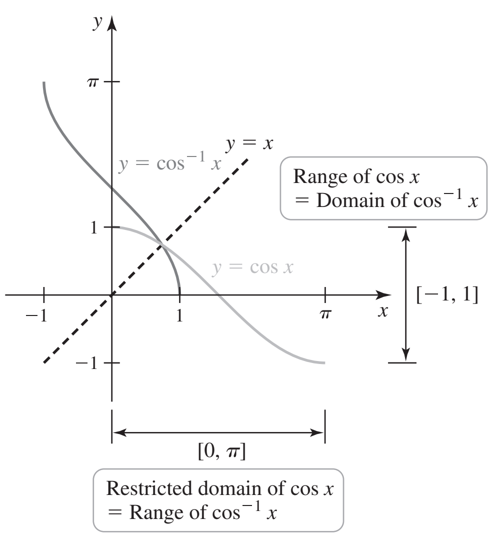


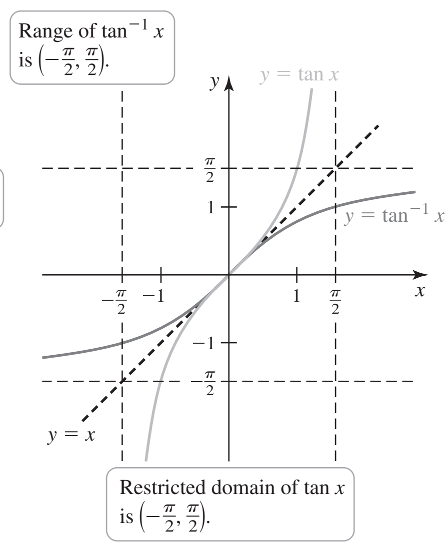
Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Graphs of Trigonometric Functions and Their Inverses







Calculus

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Calculus

Second Edition

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Preface

The second edition of *Calculus* supports a three-semester or four-quarter calculus sequence typically taken by students studying mathematics, engineering, the natural sciences, or economics. The second edition has the same goals as the first edition:

- to motivate the essential ideas of calculus with a lively narrative, demonstrating the utility of calculus with applications in diverse fields;
- to introduce new topics through concrete examples, applications, and analogies, appealing to students' intuition and geometric instincts to make calculus natural and believable; and
- once this intuitive foundation is established, to present generalizations and abstractions and to treat theoretical matters in a rigorous way.

The second edition both builds on the success and addresses the inevitable deficiencies of the first edition. We have listened to and learned from the instructors who used the first edition. They have given us wise guidance about how to make the second edition an even more effective learning tool for students and a more powerful resource for instructors. Users of the book continue to tell us that it mirrors the course they teach—and more importantly, that students actually read it! Of course, the second edition also benefits from our own experiences using the book, as well as our experiences teaching mathematics at diverse institutions over the past 30 years.

We are grateful to users of the first edition—for their courage in adopting a first edition book, for their enthusiastic response to the book, and for their invaluable advice and feedback. They deserve much of the credit for the improvements that we have made in the second edition.

New in the Second Edition

Narrative

The second edition of this book has undergone a thorough cover-to-cover polishing of the narrative, making the presentation of material even more concise and lucid. Occasionally, we discovered new ways to present material to make the exposition clearer for students and more efficient for instructors.

Figures

The figures—already dynamic and informative in the first edition—were thoroughly reviewed and revised when necessary. The figures enrich the overall spirit of the book and tell as much of the calculus story as the words do. The path-breaking interactive figures in the companion eBook have been refined, and they still represent a revolutionary way to communicate mathematics. See page xiv, eBook with Interactive Figures, for more information.

Exercises

The comprehensive 7656 exercises in the first edition were thoroughly reviewed and refined. Then 19% more basic skills and mid-level exercises were added. The exercises at the end of each section are still efficiently organized in the following categories.

- *Review Questions* begin each exercise set and check students' conceptual understanding of the essential ideas from the section.
- *Basic Skills* exercises are confidence-building problems that provide a solid foundation for the more challenging exercises to follow. Each example in the narrative is linked directly to a block of *Basic Skills* exercises via *Related Exercises* references at the end of the example solution.
- *Further Explorations* exercises expand on the *Basic Skills* exercises by challenging students to think creatively and to generalize newly acquired skills.
- *Applications* exercises connect skills developed in previous exercises to applications and modeling problems that demonstrate the power and utility of calculus.
- *Additional Exercises* are generally the most difficult and challenging problems; they include proofs of results cited in the narrative.

Each chapter concludes with a comprehensive set of *Review Exercises*.

Answers

The answers in the back of the book have been reviewed and thoroughly checked for accuracy. The reliability that we achieved in the first edition has been maintained—if not improved.

New Topics

We have added new material on Newton's method, surface area of solids of revolution, hyperbolic functions, and TNB frames. Based on our own teaching experience, we also added a brief new introductory section to the chapter on Techniques of Integration. We felt it makes sense to introduce students to some general integration strategies before diving into the standard techniques of integration by parts, partial fractions, and various substitutions.

MyMathLab

We (together with the team at Pearson) have made many improvements to the MyMathLab course for the second edition. Hundreds of new algorithmic exercises that correspond to those in the text were added to the course. Cumulative review exercises have been added, providing an opportunity for students to get "mixed practice" with important skills such as finding derivatives. New step-by-step exercises for key skills provide support for students in their first attempts at new and important problems. Real-world exercises now require that students provide units with their answer. We've added more exercises that call for student manipulation and analysis of the Interactive Figures. We have greatly increased the number of instructional videos. The graphing functionality in MyMathLab has become more sophisticated and the answer-checking algorithms are more refined.

Differential Equations

This book has a single robust section devoted to an overview of differential equations. However, for schools that require more expansive coverage of differential equations, we provide complete online chapters on both first- and second-order differential equations, available in MyMathLab as well as through the Pearson Math and Stats Resource page at **www.pearsonhighered.com/mathstatsresources**.

Pedagogical Features

Figures

Given the power of graphics software and the ease with which many students assimilate visual images, we devoted considerable time and deliberation to the figures in this book. Whenever possible, we let the figures communicate essential ideas using annotations

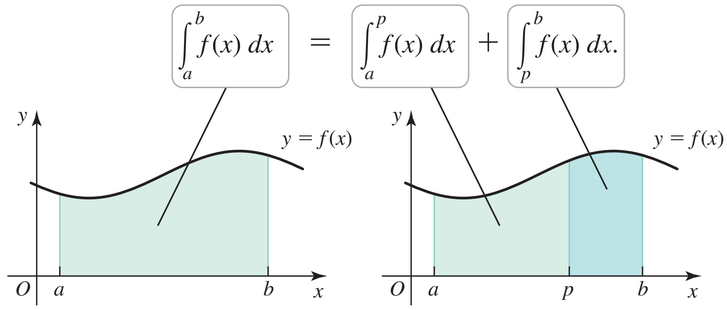

Figure 5.29

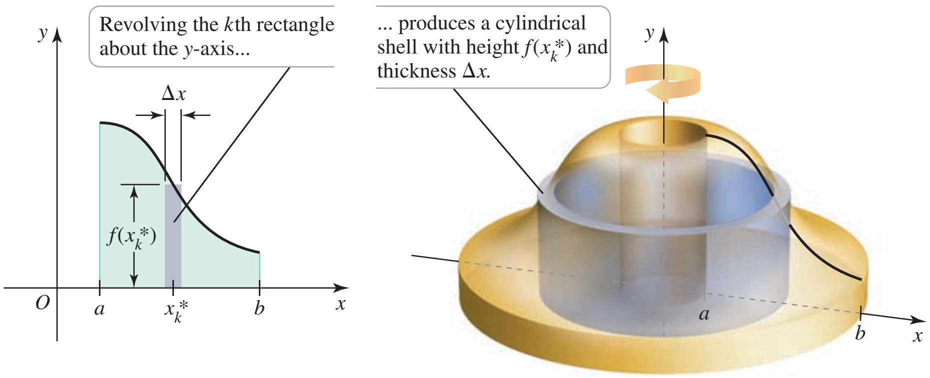

Figure 6.40

reminiscent of an instructor's voice at the board. Readers will quickly find that the figures facilitate learning in new ways.

Quick Check and *Margin Notes*

The narrative is interspersed with *Quick Check* questions that encourage students to read with pencil in hand. These questions resemble the kinds of questions instructors pose in class. Answers to the *Quick Check* questions are found at the end of the section in which they occur. *Margin Notes* offer reminders, provide insight, and clarify technical points.

Guided Projects

The *Instructor's Resource Guide and Test Bank* contains 78 *Guided Projects*. These projects allow students to work in a directed, step-by-step fashion, with various objectives: to carry out extended calculations, to derive physical models, to explore related theoretical topics, or to investigate new applications of calculus. The *Guided Projects* vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience. A list of suggested *Guided Projects* is included at the end of each chapter. Students may access the *Guided Projects* within MyMathLab.

Technology

We believe that a calculus text should help students strengthen their analytical skills and demonstrate how technology can extend (not replace) those skills. Calculators and graphing utilities are additional tools in the kit, and students must learn when and when not to use them. Our goal is to accommodate the different policies about technology that various instructors may use.

Throughout the book, exercises marked with **T** indicate that the use of technology— ranging from plotting a function with a graphing calculator to carrying out a calculation using a computer algebra system—may be needed. See page xvi for information regarding our technology resource manuals covering Maple, Mathematica and Texas Instruments graphing calculators.

eBook with Interactive Figures

The textbook is supported by a groundbreaking and award-winning electronic book, created by Eric Schulz of Walla Walla Community College. This "live book" contains the complete text of the print book plus interactive versions of approximately 700 figures. Instructors can use these interactive figures in the classroom to illustrate the important ideas of calculus, and students can explore them while they are reading the textbook. Our experience confirms that the interactive figures help build students' geometric intuition of calculus. The authors have written Interactive Figure Exercises that can be assigned via MyMathLab so that students can engage with the figures outside of class in a directed way. Additionally, the authors have created short videos, accessed through the eBook, that tell the story of key Interactive Figures. Available only within MyMathLab, the eBook provides instructors with powerful new teaching tools that expand and enrich the learning experience for students.

Content Highlights

In writing this text, we identified content in the calculus curriculum that consistently presents challenges to our students. We made organizational changes to the standard presentation of these topics or slowed the pace of the narrative to facilitate students' comprehension of material that is traditionally difficult. Two noteworthy modifications to the traditional table of contents for this course appear in the material for Calculus II and Calculus III.

Often appearing near the end of the term, the topics of sequences and series are the most challenging in Calculus II. By splitting this material into two chapters, we have given these topics a more deliberate pace and made them more accessible without adding significantly to the length of the narrative.

There is a clear and logical path through multivariate calculus, which is not apparent in many textbooks. We have carefully separated functions of several variables from vector-valued functions, so that these ideas are distinct in the minds of students. The book culminates when these two threads are joined in the last chapter, which is devoted to vector calculus.

Additional Resources

Instructor's Resource Guide and Test Bank

ISBN 0-321-95487-4 | 978-0-321-95487-9
Bernard Gillett, University of Colorado at Boulder

This guide represents significant contributions by the textbook authors and contains a variety of classroom support materials for instructors.

- Seventy-eight *Guided Projects*, correlated to specific chapters of the text, can be assigned to students for individual or group work. The *Guided Projects* vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience.
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Calculus

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1

Functions

1.1 Review of Functions
1.2 Representing Functions
1.3 Trigonometric Functions

Chapter Preview Mathematics is a language with an alphabet, a vocabulary, and many rules. Before beginning your calculus journey, you should be familiar with the elements of this language. Among these elements are algebra skills; the notation and terminology for various sets of real numbers; and the descriptions of lines, circles, and other basic sets in the coordinate plane. A review of this material is found in Appendix A. This chapter begins with the fundamental concept of a function and then presents some of the functions needed for calculus: polynomials, rational functions, algebraic functions, and the trigonometric functions. (Logarithmic, exponential, and inverse functions are introduced in Chapter 7.) Before you begin studying calculus, it is important that you master the ideas in this chapter.

1.1 Review of Functions

Everywhere around us we see relationships among quantities, or **variables**. For example, the consumer price index changes in time and the temperature of the ocean varies with latitude. These relationships can often be expressed by mathematical objects called *functions*. Calculus is the study of functions, and because we use functions to describe the world around us, calculus is a universal language for human inquiry.

DEFINITION Function

A **function** f is a rule that assigns to each value x in a set D a *unique* value denoted $f(x)$. The set D is the **domain** of the function. The **range** is the set of all values of $f(x)$ produced as x varies over the entire domain (Figure 1.1).

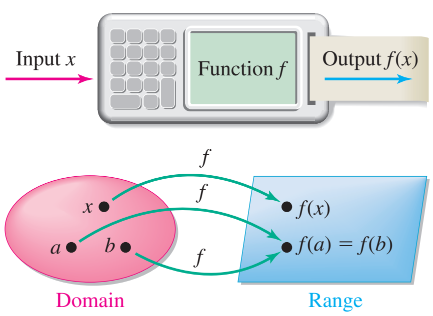

Figure 1.1

➤ If the domain is not specified, we take it to be the set of all values of x for which f is defined. We will see shortly that the domain and range of a function may be restricted by the context of the problem.

The **independent variable** is the variable associated with the domain; the **dependent variable** belongs to the range. The **graph** of a function f is the set of all points (x, y) in the xy-plane that satisfies the equation $y = f(x)$. The **argument** of a function is the expression on which the function works. For example, x is the argument when we write $f(x)$. Similarly, 2 is the argument in $f(2)$ and $x^2 + 4$ is the argument in $f(x^2 + 4)$.

QUICK CHECK 1 If $f(x) = x^2 - 2x$, find $f(-1)$, $f(x^2)$, $f(t)$, and $f(p - 1)$. ◄

The requirement that a function assigns a *unique* value of the dependent variable to each value in the domain is expressed in the vertical line test (Figure 1.2a). For example, the outside temperature as it varies over the course of a day is a function of time (Figure 1.2b).

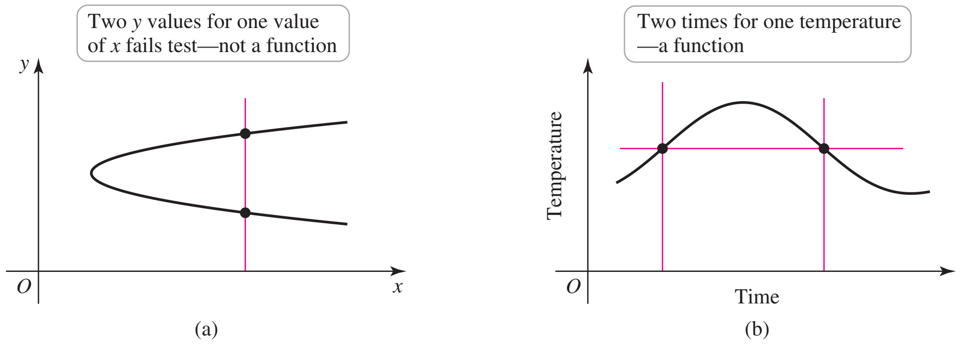

Figure 1.2

Vertical Line Test

A graph represents a function if and only if it passes the **vertical line test**: Every vertical line intersects the graph at most once. A graph that fails this test does not represent a function.

➤ A set of points or a graph that does *not* correspond to a function represents a **relation** between the variables. All functions are relations, but not all relations are functions.

EXAMPLE 1 Identifying functions State whether each graph in Figure 1.3 represents a function.

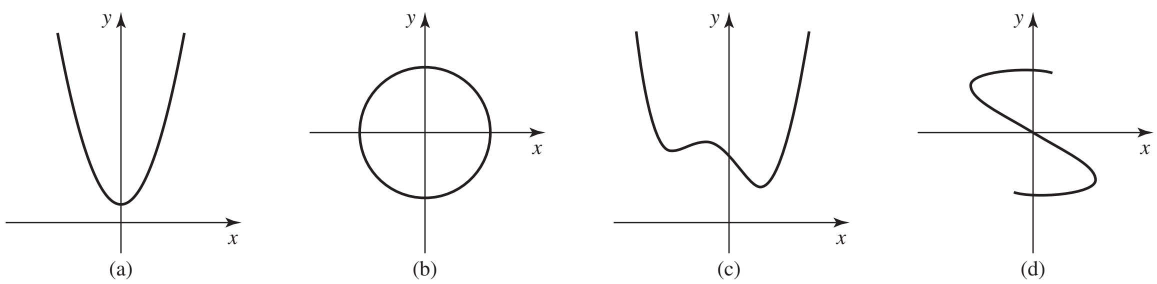

Figure 1.3

SOLUTION The vertical line test indicates that only graphs (a) and (c) represent functions. In graphs (b) and (d), there are vertical lines that intersect the graph more than once. Equivalently, there are values of x that correspond to more than one value of y. Therefore, graphs (b) and (d) do not pass the vertical line test and do not represent functions. *Related Exercises 11–12* ◄

➤ A window of $[a, b] \times [c, d]$ means $a \le x \le b$ and $c \le y \le d$.

EXAMPLE 2 Domain and range Graph each function with a graphing utility using the given window. Then state the domain and range of the function.

a. $y = f(x) = x^2 + 1; \quad [-3, 3] \times [-1, 5]$

b. $z = g(t) = \sqrt{4 - t^2}; \quad [-3, 3] \times [-1, 3]$

c. $w = h(u) = \dfrac{1}{u - 1}; \quad [-3, 5] \times [-4, 4]$

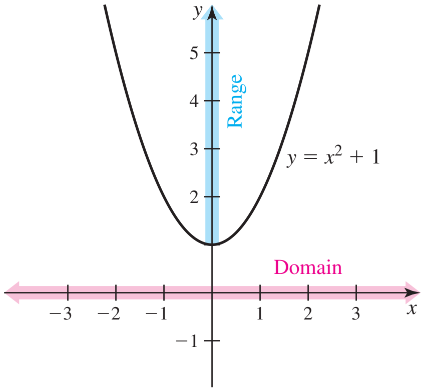

Figure 1.4

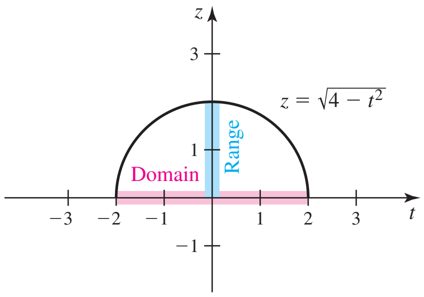

Figure 1.5

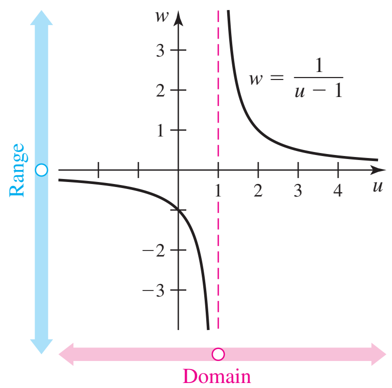

Figure 1.6

➤ The dashed vertical line $u = 1$ in Figure 1.6 indicates that the graph of $w = h(u)$ approaches a *vertical asymptote* as u approaches 1 and that w becomes large in magnitude for u near 1.

SOLUTION

a. Figure 1.4 shows the graph of $f(x) = x^2 + 1$. Because f is defined for all values of x, its domain is the set of all real numbers, written $(-\infty, \infty)$ or $\mathbb{R}$. Because $x^2 \geq 0$ for all x, it follows that $x^2 + 1 \geq 1$ and the range of f is $[1, \infty)$.

b. When n is even, functions involving nth roots are defined provided the quantity under the root is nonnegative (additional restrictions may also apply). In this case, the function g is defined provided $4 - t^2 \geq 0$, which means $t^2 \leq 4$, or $-2 \leq t \leq 2$. Therefore, the domain of g is $[-2, 2]$. By the definition of the square root, the range consists only of nonnegative numbers. When $t = 0$, z reaches its maximum value of $g(0) = \sqrt{4} = 2$, and when $t = \pm 2$, z attains its minimum value of $g(\pm 2) = 0$. Therefore, the range of g is $[0, 2]$ (Figure 1.5).

c. The function h is undefined at $u = 1$, so its domain is $\{u: u \neq 1\}$, and the graph does not have a point corresponding to $u = 1$. We see that w takes on all values except 0; therefore, the range is $\{w: w \neq 0\}$. A graphing utility does *not* represent this function accurately if it shows the vertical line $u = 1$ as part of the graph (Figure 1.6).

Related Exercises 13–20 ◄

EXAMPLE 3 Domain and range in context At time $t = 0$, a stone is thrown vertically upward from the ground at a speed of 30 m/s. Its height above the ground in meters (neglecting air resistance) is approximated by the function $h = f(t) = 30t - 5t^2$, where t is measured in seconds. Find the domain and range of f in the context of this particular problem.

SOLUTION Although f is defined for all values of t, the only relevant times are between the time the stone is thrown ($t = 0$) and the time it strikes the ground, when $h = f(t) = 0$. Solving the equation $h = 30t - 5t^2 = 0$, we find that

$$\begin{aligned} 30t - 5t^2 &= 0 && \\ 5t(6 - t) &= 0 && \text{Factor.} \\ 5t = 0 \quad \text{or} \quad 6 - t &= 0 && \text{Set each factor equal to 0.} \\ t = 0 \quad \text{or} \quad t &= 6. && \text{Solve.} \end{aligned}$$

Therefore, the stone leaves the ground at $t = 0$ and returns to the ground at $t = 6$. An appropriate domain that fits the context of this problem is $\{t: 0 \leq t \leq 6\}$. The range consists of all values of $h = 30t - 5t^2$ as t varies over $[0, 6]$. The largest value of h occurs when the stone reaches its highest point at $t = 3$ (halfway through its flight), which is $h = f(3) = 45$. Therefore, the range is $[0, 45]$. These observations are confirmed by the graph of the height function (Figure 1.7). Note that this graph is *not* the trajectory of the stone; the stone moves vertically.

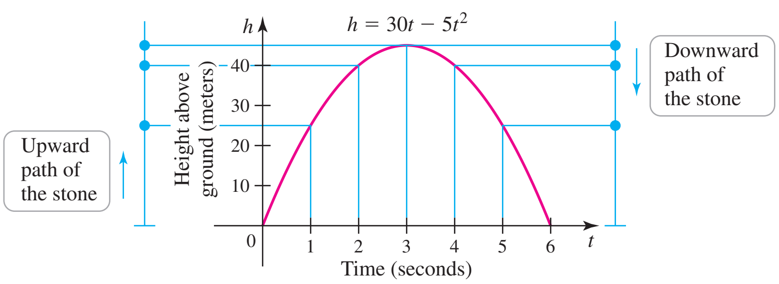

Figure 1.7

Related Exercises 21–24 ◄

QUICK CHECK 2 State the domain and range of $f(x) = (x^2 + 1)^{-1}$. ◄

Composite Functions

Functions may be combined using sums $(f + g)$, differences $(f - g)$, products (fg), or quotients (f/g). The process called *composition* also produces new functions.

➤ In the composition $y = f(g(x))$, f is the *outer function* and g is the *inner function*.

DEFINITION Composite Functions

Given two functions f and g, the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. It is evaluated in two steps: $y = f(u)$, where $u = g(x)$. The domain of $f \circ g$ consists of all x in the domain of g such that $u = g(x)$ is in the domain of f (Figure 1.8).

➤ You have now seen three different notations for intervals on the real number line, all of which will be used throughout the book:
- $[-2, 3)$ is an example of interval notation,
- $-2 \le x < 3$ is inequality notation, and
- $\{x: -2 \le x < 3\}$ is set notation.

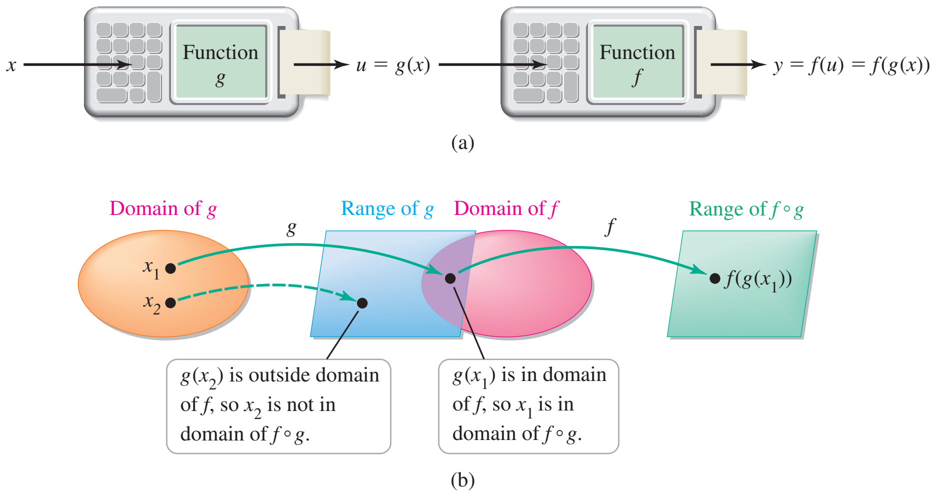

Figure 1.8

EXAMPLE 4 Composite functions and notation Let $f(x) = 3x^2 - x$ and $g(x) = 1/x$. Simplify the following expressions.

a. $f(5p + 1)$ **b.** $g(1/x)$ **c.** $f(g(x))$ **d.** $g(f(x))$

SOLUTION In each case, the functions work on their arguments.

a. The argument of f is $5p + 1$, so

$$f(5p + 1) = 3(5p + 1)^2 - (5p + 1) = 75p^2 + 25p + 2.$$

b. Because g requires taking the reciprocal of the argument, we take the reciprocal of $1/x$ and find that $g(1/x) = 1/(1/x) = x$.

c. The argument of f is $g(x)$, so

$$f(g(x)) = f\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right) = \frac{3}{x^2} - \frac{1}{x} = \frac{3 - x}{x^2}.$$

➤ Examples 4c and 4d demonstrate that, in general,

$$f(g(x)) \ne g(f(x)).$$

d. The argument of g is $f(x)$, so

$$g(f(x)) = g(3x^2 - x) = \frac{1}{3x^2 - x}.$$

Related Exercises 25–36 ◄

EXAMPLE 5 Working with composite functions Identify possible choices for the inner and outer functions in the following composite functions. Give the domain of the composite function.

a. $h(x) = \sqrt{9x - x^2}$ **b.** $h(x) = \dfrac{2}{(x^2 - 1)^3}$

SOLUTION

➤ Techniques for solving inequalities are discussed in Appendix A.

a. An obvious outer function is $f(x) = \sqrt{x}$, which works on the inner function $g(x) = 9x - x^2$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of x such that $9x - x^2 \ge 0$. Solving this inequality gives $\{x: 0 \le x \le 9\}$ as the domain of $f \circ g$.

b. A good choice for an outer function is $f(x) = 2/x^3 = 2x^{-3}$, which works on the inner function $g(x) = x^2 - 1$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of $g(x)$ such that $g(x) \neq 0$, which is $\{x: x \neq \pm 1\}$.

Related Exercises 37–40 ◄

EXAMPLE 6 **More composite functions** Given $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$, find (a) $g \circ f$ and (b) $g \circ g$, and their domains.

SOLUTION

a. We have

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x}) = \underbrace{(\sqrt[3]{x})}_{f(x)}{}^2 - \underbrace{\sqrt[3]{x}}_{f(x)} - 6 = x^{2/3} - x^{1/3} - 6.$$

Because the domains of f and g are $(-\infty, \infty)$, the domain of $f \circ g$ is also $(-\infty, \infty)$.

b. In this case, we have the composition of two polynomials:

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(x^2 - x - 6) \\ &= \underbrace{(x^2 - x - 6)^2}_{g(x)} - \underbrace{(x^2 - x - 6)}_{g(x)} - 6 \\ &= x^4 - 2x^3 - 12x^2 + 13x + 36.\end{aligned}$$

The domain of the composition of two polynomials is $(-\infty, \infty)$.

Related Exercises 41–54 ◄

QUICK CHECK 3 If $f(x) = x^2 + 1$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$. ◄

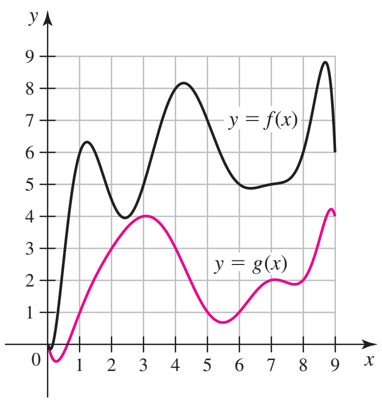

Figure 1.9

EXAMPLE 7 **Using graphs to evaluate composite functions** Use the graphs of f and g in Figure 1.9 to find the following values.

a. $f(g(3))$ **b.** $g(f(3))$ **c.** $f(f(4))$ **d.** $f(g(f(8)))$

SOLUTION

a. The graphs indicate that $g(3) = 4$ and $f(4) = 8$, so $f(g(3)) = f(4) = 8$.

b. We see that $g(f(3)) = g(5) = 1$. Observe that $f(g(3)) \neq g(f(3))$.

c. In this case, $f(\underbrace{f(4)}_{8}) = f(8) = 6$.

d. Starting on the inside,

$$f(g(\underbrace{f(8)}_{6})) = f(\underbrace{g(6)}_{1}) = f(1) = 6.$$

Related Exercises 55–56 ◄

EXAMPLE 8 **Using a table to evaluate composite functions** Use the function values in the table to evaluate the following composite functions.

a. $(f \circ g)(0)$ **b.** $g(f(-1))$ **c.** $f(g(g(-1)))$

x	-2	-1	0	1	2
$f(x)$	0	1	3	4	2
$g(x)$	-1	0	-2	-3	-4

SOLUTION

a. Using the table, we see that $g(0) = -2$ and $f(-2) = 0$. Therefore, $(f \circ g)(0) = 0$.

b. Because $f(-1) = 1$ and $g(1) = -3$, it follows that $g(f(-1)) = -3$.

c. Starting with the inner function,

$$f(g(\underbrace{g(-1)}_{0})) = f(\underbrace{g(0)}_{-2}) = f(-2) = 0.$$

Related Exercises 55–56 ◀

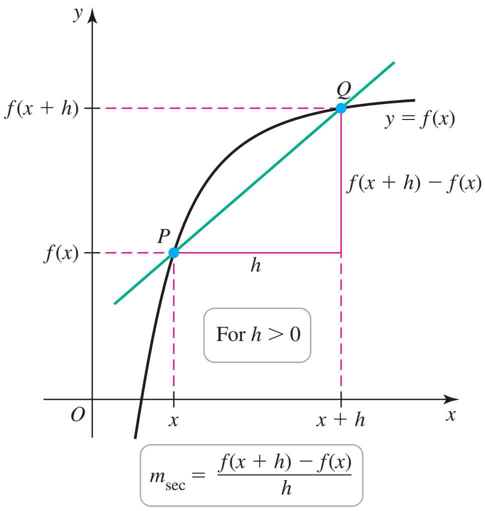

Figure 1.10

Secant Lines and the Difference Quotient

As you will see shortly, slopes of lines and curves play a fundamental role in calculus. Figure 1.10 shows two points $P(x, f(x))$ and $Q(x + h, f(x + h))$ on the graph of $y = f(x)$ in the case that $h > 0$. A line through any two points on a curve is called a **secant line**; its importance in the study of calculus is explained in Chapters 2 and 3. For now, we focus on the slope of the secant line through P and Q, which is denoted m_{sec} and is given by

$$m_{\text{sec}} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{f(x + h) - f(x)}{h}.$$

The slope formula $\dfrac{f(x + h) - f(x)}{h}$ is also known as a **difference quotient**, and it can be expressed in several ways depending on how the coordinates of P and Q are labeled. For example, given the coordinates $P(a, f(a))$ and $Q(x, f(x))$ (Figure 1.11), the difference quotient is

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}.$$

We interpret the slope of the secant line in this form as the **average rate of change** of f over the interval $[a, x]$.

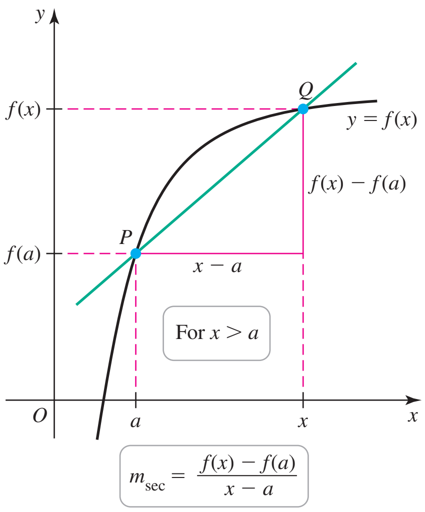

Figure 1.11

EXAMPLE 9 **Working with difference quotients**

a. Simplify the difference quotient $\dfrac{f(x + h) - f(x)}{h}$, for $f(x) = 3x^2 - x$.

b. Simplify the difference quotient $\dfrac{f(x) - f(a)}{x - a}$, for $f(x) = x^3$.

SOLUTION

a. First note that $f(x + h) = 3(x + h)^2 - (x + h)$. We substitute this expression into the difference quotient and simplify:

$$\begin{aligned}
\frac{f(x + h) - f(x)}{h} &= \frac{\overbrace{3(x + h)^2 - (x + h)}^{f(x+h)} - \overbrace{(3x^2 - x)}^{f(x)}}{h} \\
&= \frac{3(x^2 + 2xh + h^2) - (x + h) - (3x^2 - x)}{h} && \text{Expand } (x + h)^2. \\
&= \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h} && \text{Distribute.} \\
&= \frac{6xh + 3h^2 - h}{h} && \text{Simplify.} \\
&= \frac{h(6x + 3h - 1)}{h} = 6x + 3h - 1. && \text{Factor and simplify.}
\end{aligned}$$

➤ Treat $f(x + h)$ like the composition $f(g(x))$, where $x + h$ plays the role of $g(x)$. It may help to establish a pattern in your mind before evaluating $f(x + h)$. For instance, using the function in Example 9a, we have

$$f(x) = 3x^2 - x;$$
$$f(12) = 3 \cdot 12^2 - 12;$$
$$f(b) = 3b^2 - b;$$
$$f(\text{math}) = 3 \cdot \text{math}^2 - \text{math};$$

therefore,

$$f(x + h) = 3(x + h)^2 - (x + h).$$

➤ Some useful factoring formulas:

1. Difference of perfect squares:
   $x^2 - y^2 = (x - y)(x + y)$.
2. Sum of perfect squares: $x^2 + y^2$ does not factor over the real numbers.
3. Difference of perfect cubes:
   $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.
4. Sum of perfect cubes:
   $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.

b. The factoring formula for the difference of perfect cubes is needed:

$$\begin{aligned}\frac{f(x) - f(a)}{x - a} &= \frac{x^3 - a^3}{x - a}\\ &= \frac{(x - a)(x^2 + ax + a^2)}{x - a} && \text{Factoring formula}\\ &= x^2 + ax + a^2. && \text{Simplify.}\end{aligned}$$

Related Exercises 57–66 ◄

EXAMPLE 10 Interpreting the slope of the secant line Sound intensity I, measured in watts per square meter (W/m^2), at a point r meters from a sound source with acoustic power P is given by $I(r) = \frac{P}{4\pi r^2}$.

a. Find the sound intensity at two points $r_1 = 10$ m and $r_2 = 15$ m from a sound source with power $P = 100$ W. Then find the slope of the secant line through the points $(10, I(10))$ and $(15, I(15))$ on the graph of the intensity function and interpret the result.

b. Find the slope of the secant line through any two points $(r_1, I(r_1))$ and $(r_2, I(r_2))$ on the graph of the intensity function with acoustic power P.

SOLUTION

a. The sound intensity 10 m from the source is $I(10) = \frac{100 \text{ W}}{4\pi(10 \text{ m})^2} = \frac{1}{4\pi}\,\text{W/m}^2$. At 15 m, the intensity is $I(15) = \frac{100 \text{ W}}{4\pi(15 \text{ m})^2} = \frac{1}{9\pi}\,\text{W/m}^2$. To find the slope of the secant line (Figure 1.12), we compute the change in intensity divided by the change in distance:

$$m_{\text{sec}} = \frac{I(15) - I(10)}{15 - 10} = \frac{\frac{1}{9\pi} - \frac{1}{4\pi}}{5} = -\frac{1}{36\pi} \approx -0.0088 \text{ W/m}^2 \text{ per meter.}$$

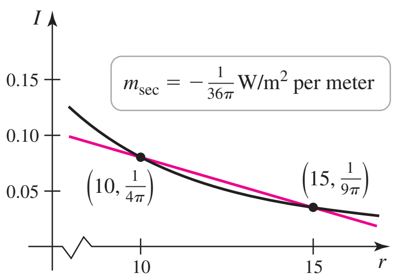

Figure 1.12

The units provide a clue to the physical meaning of the slope: It measures the average rate at which the intensity changes as one moves from 10 m to 15 m away from the sound source. In this case, because the slope of the secant line is negative, the intensity *decreases* (slowly) at an average rate of $1/(36\pi)$ W/m^2 per meter.

b.

$$\begin{aligned}m_{\text{sec}} = \frac{I(r_2) - I(r_1)}{r_2 - r_1} &= \frac{\frac{P}{4\pi r_2^2} - \frac{P}{4\pi r_1^2}}{r_2 - r_1} && \text{Evaluate } I(r_2) \text{ and } I(r_1).\\ &= \frac{\frac{P}{4\pi}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)}{r_2 - r_1} && \text{Factor.}\\ &= \frac{P}{4\pi}\left(\frac{r_1^2 - r_2^2}{r_1^2 r_2^2}\right)\frac{1}{r_2 - r_1} && \text{Simplify.}\\ &= \frac{P}{4\pi} \cdot \frac{(r_1 - r_2)(r_1 + r_2)}{r_1^2 r_2^2} \cdot \frac{1}{-(r_1 - r_2)} && \text{Factor.}\\ &= -\frac{P(r_1 + r_2)}{4\pi r_1^2 r_2^2} && \text{Cancel and simplify.}\end{aligned}$$

This result is the average rate at which the sound intensity changes over an interval $[r_1, r_2]$. Because $r_1 > 0$ and $r_2 > 0$, we see that m_{sec} is always negative. Therefore, the sound intensity $I(r)$ decreases as r increases, for $r > 0$.

Related Exercises 67–70 ◄

Symmetry

The word *symmetry* has many meanings in mathematics. Here we consider symmetries of graphs and the relations they represent. Taking advantage of symmetry often saves time and leads to insights.

> **DEFINITION Symmetry in Graphs**
>
> A graph is **symmetric with respect to the *y*-axis** if whenever the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph. This property means that the graph is unchanged when reflected across the y-axis (Figure 1.13a).
>
> A graph is **symmetric with respect to the *x*-axis** if whenever the point (x, y) is on the graph, the point $(x, -y)$ is also on the graph. This property means that the graph is unchanged when reflected across the x-axis (Figure 1.13b).
>
> A graph is **symmetric with respect to the origin** if whenever the point (x, y) is on the graph, the point $(-x, -y)$ is also on the graph (Figure 1.13c). Symmetry about both the x- and y-axes implies symmetry about the origin, but not vice versa.

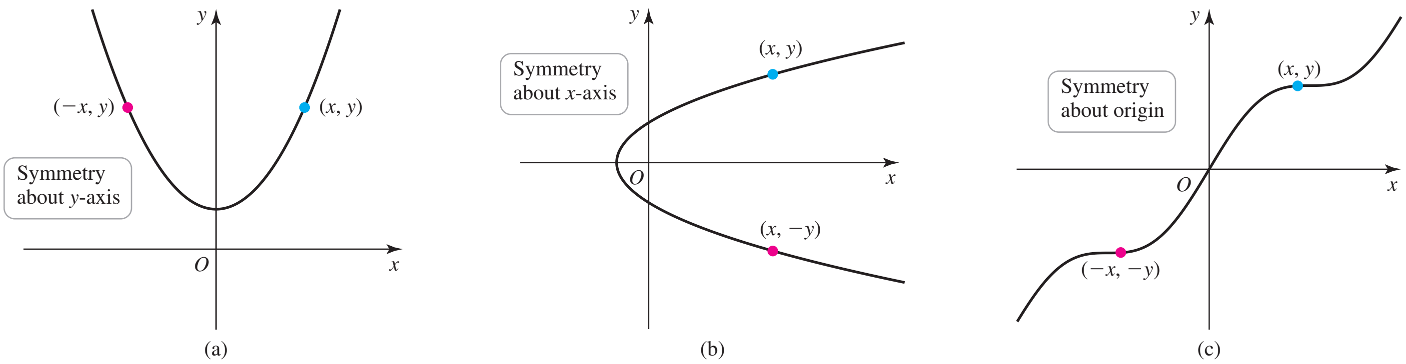

Figure 1.13

> **DEFINITION Symmetry in Functions**
>
> An **even function** f has the property that $f(-x) = f(x)$, for all x in the domain. The graph of an even function is symmetric about the y-axis.
>
> An **odd function** f has the property that $f(-x) = -f(x)$, for all x in the domain. The graph of an odd function is symmetric about the origin.

Polynomials consisting of only even powers of the variable (of the form x^{2n}, where n is a nonnegative integer) are even functions. Polynomials consisting of only odd powers of the variable (of the form x^{2n+1}, where n is a nonnegative integer) are odd functions.

QUICK CHECK 4 Explain why the graph of a nonzero function is never symmetric with respect to the x-axis. ◄

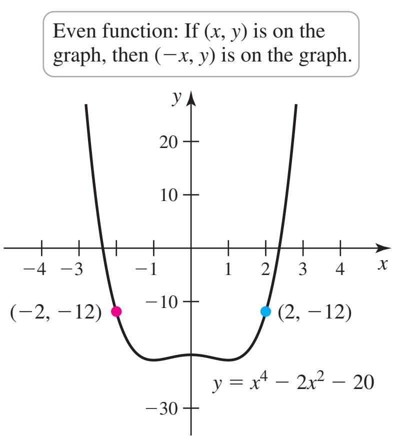

Figure 1.14

EXAMPLE 11 Identifying symmetry in functions Identify the symmetry, if any, in the following functions.

a. $f(x) = x^4 - 2x^2 - 20$ **b.** $g(x) = x^3 - 3x + 1$ **c.** $h(x) = \dfrac{1}{x^3 - x}$

SOLUTION

a. The function f consists of only even powers of x (where $20 = 20 \cdot 1 = 20x^0$ and x^0 is considered an even power). Therefore, f is an even function (Figure 1.14). This fact is verified by showing that $f(-x) = f(x)$:

$$f(-x) = (-x)^4 - 2(-x)^2 - 20 = x^4 - 2x^2 - 20 = f(x).$$

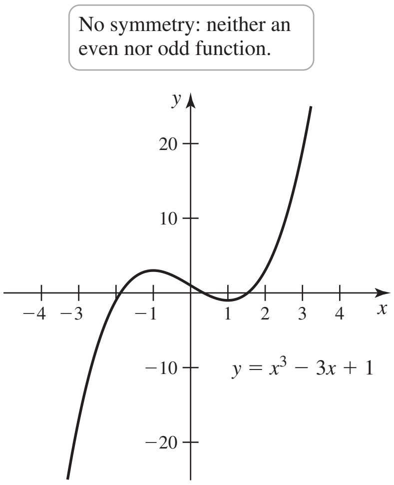

Figure 1.15

➤ The symmetry of compositions of even and odd functions is considered in Exercises 95–101.

b. The function g consists of two odd powers and one even power (again, $1 = x^0$ is an even power). Therefore, we expect that g has no symmetry about the y-axis or the origin (Figure 1.15). Note that

$$g(-x) = (-x)^3 - 3(-x) + 1 = -x^3 + 3x + 1,$$

so $g(-x)$ equals neither $g(x)$ nor $-g(x)$; therefore, g has no symmetry.

c. In this case, h is a composition of an odd function $f(x) = 1/x$ with an odd function $g(x) = x^3 - x$. Note that

$$h(-x) = \frac{1}{(-x)^3 - (-x)} = -\frac{1}{x^3 - x} = -h(x).$$

Because $h(-x) = -h(x)$, h is an odd function (Figure 1.16).

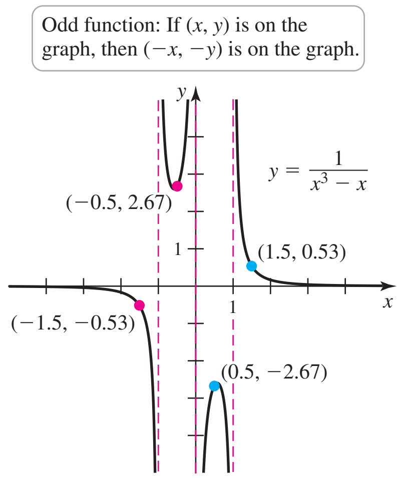

Figure 1.16

Related Exercises 71–80 ◄

SECTION 1.1 EXERCISES

Review Questions

1. Use the terms *domain, range, independent variable*, and *dependent variable* to explain how a function relates one variable to another variable.

2. Is the independent variable of a function associated with the domain or range? Is the dependent variable associated with the domain or range?

3. Explain how the vertical line test is used to detect functions.

4. If $f(x) = 1/(x^3 + 1)$, what is $f(2)$? What is $f(y^2)$?

5. Which statement about a function is true? (i) For each value of x in the domain, there corresponds one unique value of y in the range; (ii) for each value of y in the range, there corresponds one unique value of x in the domain. Explain.

6. If $f(x) = \sqrt{x}$ and $g(x) = x^3 - 2$, find the compositions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$.

7. Suppose f and g are even functions with $f(2) = 2$ and $g(2) = -2$. Evaluate $f(g(2))$ and $g(f(-2))$.

8. Explain how to find the domain of $f \circ g$ if you know the domain and range of f and g.

9. Sketch a graph of an even function f and state how $f(x)$ and $f(-x)$ are related.

10. Sketch a graph of an odd function f and state how $f(x)$ and $f(-x)$ are related.

Basic Skills

11–12. Vertical line test *Decide whether graphs A, B, or both represent functions.*

11.